

LNF - 64/14  
23 Aprile 1964.

E. Celeghini and R. Gatto: POSSIBLE METHOD TO DETERMINE  
THE  $\eta^0$  LIFETIME.

(Nota interna: n. 238)

Nota interna: n. 238  
23 Aprile 1964.

E. Celeghini and R. Gatto: POSSIBLE METHOD TO DETERMINE  
THE  $\eta^0$  LIFETIME. -

1. Very little is known at present on the  $\eta^0$  lifetime. The experimental upper limit on the  $\eta^0$  width only tells us that  $\eta^0$  lives longer than  $\sim 10^{-22}$  sec<sup>(1)</sup>. The  $\eta^0$  quantum numbers ( $J^P = 0^-, I^G = 0^+$ ) imply that  $\eta^0$  decays electromagnetically. The expected lifetime is thus much longer. The only theoretical prediction, at this moment, on the  $\eta^0$  lifetime is based on the comparison<sup>(2)</sup> of the  $\eta^0 \rightarrow 2\gamma$  to the  $\pi^0 \rightarrow 2\gamma$  amplitude, on the basis of full invariance of strong interactions under  $SU_3$ . The predicted  $\eta^0 \rightarrow 2\gamma$  rate is  $w(\eta^0 \rightarrow 2\gamma) = 22 w(\pi^0 \rightarrow 2\gamma)$ . With a  $\pi^0 \rightarrow 2\gamma$  lifetime of  $1.3 \times 10^{-16}$  sec one has a rate of  $\eta^0 \rightarrow 2\gamma$  of  $1.7 \times 10^{17}$  sec<sup>-1</sup>. According to Puppi<sup>(3)</sup> the expected branching ratio into  $\eta^0 \rightarrow 2\gamma$  is about 38%.

Proposals to determine the  $\eta^0$  lifetime have already been advanced. They all bear on very difficult experiments. Bellettini et al. have proposed to measure the  $\eta^0$  Primakoff effect on a heavy nucleus from high energy (4 - 5 BeV) photon beams<sup>(4)</sup>. The  $\eta^0$  Primakoff effect consists in the coherent photoproduction of a  $\eta^0$  on the Coulomb field of a nucleus. Much care must be used in discriminating against coherent nuclear photoproduction. Contogouris and Verganelakis have proposed proton Compton scattering with polarized photons to compare the relative importance of the  $\eta^0$  intermediate state to the  $2\pi$  intermediate s-wave<sup>(5)</sup>. Celeghini and Gatto<sup>(6)</sup> have proposed the electron positron colliding reactions  $e^+ + e^- \rightarrow \eta^0 + \gamma$  and  $e^+ + e^- \rightarrow \eta^0 + \pi^+ + \pi^-$  to obtain a direct information on the  $\eta^0 \rightarrow 2\gamma$  and  $\eta^0 \rightarrow \pi^+ + \pi^- + \gamma$  vertices. According to Puppi  $\eta^0 \rightarrow \pi^+ + \pi^- + \gamma$  occurs only in about 6% of the times<sup>(3)</sup>. In spite of this fact  $e^+ + e^- \rightarrow \eta^0 + \pi^+ + \pi^-$  seems to be particularly promising for a determination of the  $\eta^0$  lifetime because of

the broad maximum associated with the emergence of the two pions in a resonant p-wave.

2. Puppi has pointed out the convenience of peripheral production processes of the  $\eta + \pi$  system for a determination of the  $\eta^0$  lifetime<sup>(3)</sup>. In this note we discuss in particular the process

$$(1) \quad \gamma + p \rightarrow n + \eta^0 + \pi^+$$

in view of the possible utilization of its peripheral contribution of fig. 1, for

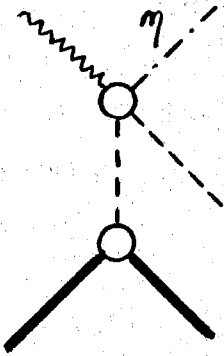


Fig. 1

a determination of the  $\eta^0$  lifetime. Our conclusion is that, for experiments performed above 2 - 3 BeV incident photon momentum, the peripheral contribution may in fact be easily detected and provide an excellent determination of the  $\eta^0$  lifetime.

In reaction (1) we call:  $k_\mu$ ,  $q_\mu^{(2)}$ , and  $q_\mu^{(3)}$  the four-momenta of the initial photon, of the  $\eta^0$  and of the final pion;  $p_\mu^{(1)}$  and  $p_\mu^{(2)}$  the four-momenta of the initial and final nucleon respectively. We define the  $\gamma\pi \rightarrow \eta\pi$  amplitude, occurring in the upper vertex of fig. 1, as

$$(2) \quad -iG(s^2, t^2) \epsilon_{\lambda\mu\nu\rho} q_\lambda^{(1)} q_\mu^{(2)} q_\nu^{(3)} \epsilon_\rho$$

where  $q_\mu^{(1)}$  is the intermediate pion four-momentum,  $\epsilon_\mu$  is the photon polarization vector and  $G(s^2, t^2)$  is a form factor depending on  $s^2 = -(q^{(1)} - q^{(3)})^2$  and  $t^2 = -(q_2 + q_3)^2$ . We assume that the known vector meson resonances ( $\rho$  and  $\omega$ ) are the dominant intermediate states affecting the form of  $G$ . From I-spin and G-parity one sees the  $\omega$  pole does not contribute in any channel and the  $\rho$  pole only contributes in the  $s$  channel. We thus write

$$(3) \quad G = \frac{f}{(s^2 - m_\rho^2) + i\Gamma}$$

with  $\Gamma = \gamma \left( \frac{1}{4} s^2 - m_\rho^2 \right)^{3/2}$ , with  $\gamma = 0.4 m_\rho^{-1}$ . The value of  $|f|^2$  can be related to the  $\eta^0 \rightarrow 2\pi$  lifetime by the following procedure. We calculate, with the amplitude (2) and using (3), the rate of  $\eta^0 \rightarrow \pi^+ + \pi^- + \gamma$ . We find

$$(4) \quad w(\eta^0 \rightarrow \pi^+ + \pi^- + \gamma) = |f|^2 m_\eta^3 \times 3.03 \times 10^{-8}$$

The ratio  $R = (\eta^0 \rightarrow \pi^+ + \pi^- + \gamma) / (\eta^0 \rightarrow 2\gamma)$  is already approximately known, and it will be better determined in the next future<sup>(7)</sup>. Following Puppi we adopt  $R = 0.16$ <sup>(3)</sup>. Using the above unitary symmetry prediction for the  $\eta^0 \rightarrow 2\gamma$  rate,  $w(\eta^0 \rightarrow 2\gamma) = 1.7 \times 10^{17} \text{ sec}^{-1}$ , we obtain  $|f|^2 = 1.42 \times 10^{-27} \text{ cm}^2$ . With a higher  $R$  or a higher  $w(\eta^0 \rightarrow 2\gamma)$ ,  $|f|^2$  would become proportionally larger.

The peripheral (fig. 1) contribution to  $\gamma + p \rightarrow n + \eta^0 + \pi^+$  gives

$$(5) \quad \frac{\partial^2 \sigma}{\partial \Delta^2 \partial \omega^2} = \frac{g^2}{128 (2\pi)^3} \frac{f^2}{(M^2 - W^2)^2} \frac{\Delta^2}{(\omega^2 - \Delta^2)(m_\pi^2 - \Delta^2)} \left\{ 2\omega^2 v^2 + \right. \\ \left. + \left[ -(\omega^2 + m_\eta^2 - m_\pi^2)\Delta^2 + (\omega^2 + 2m_p^2 - m_\eta^2 - m_\pi^2)\omega^2 \right] \log \frac{m_p^2 - u^2 - v^2}{m_p^2 - u^2 + v^2} + \right. \\ \left. + 2 \left[ +m_\eta^2 \Delta^4 - (m_p^2 \omega^2 + m_\eta^2 \omega^2 + m_p^2 m_\eta^2 - m_p^2 m_\pi^2 - m_\eta^4 + m_\eta^2 m_\pi^2) \Delta^2 + \right. \right. \\ \left. \left. + (m_p^2 \omega^2 + m_p^4 + m_\eta^2 m_\pi^2 - m_p^2 m_\eta^2 - m_p^2 m_\pi^2) \omega^2 \right] \frac{v^2}{(m_p^2 - u^2)^2 - v^2} \right\}$$

where:  $\Delta^2 = -(p^{(1)} - p^{(2)})^2$  is the squared momentum-transfer,  $\omega^2 = -(q^{(2)} + q^{(3)})^2$  is the squared c. m. energy of the produced  $\eta + \pi$  system,  $W^2 = -(p^{(1)} + K)^2$  is the squared c. m. energy of the reaction, and  $u^2$  and  $v^2$  are defined as

$$(6) \quad u^2 = m_\pi^2 + \Delta^2 - \frac{\omega^2 + \Delta^2}{2\omega^2} (\omega^2 + m_\pi^2 - m_\eta^2)$$

$$(7) \quad v^2 = \frac{\omega^2 - \Delta^2}{2\omega^2} \left[ (\omega^2 + m_\pi^2 - m_\eta^2)^2 - 4m_\pi^2 \omega^2 \right]^{1/2}$$

We have integrated Eq. (5), between the kinematic limits, to obtain the corresponding total cross-section. The result is reported in fig. 2, full-line curve, assuming for  $|f|^2$  the above value of  $1.42 \times 10^{-27} \text{ cm}^2$ . If  $w(\eta^0 \rightarrow 2\gamma)$  is increased over the unitary symmetry prediction of  $1.7 \times 10^{17} \text{ sec}^{-1}$ , or if  $R$  is increased the cross-section of fig. 2 (full-line) must be proportionally increased.

3. We now consider another contribution to reaction (1) that we expect to be a dominant one, in order to draw some conclusion as to the relative weight of the peripheral graph of fig. 1. We shall examine in detail another

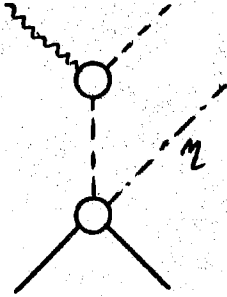


Fig. 3

peripheral mechanism based on the possible excitation of the second nucleon resonance ( $N^{**}$  at 1510 MeV, with  $I = 1/2$  and  $J^P = 3/2^-$ ). The cross-section corresponding to the peripheral mechanism of fig. 3 can be written as

$$(8) \quad \sigma = \frac{\alpha}{2\pi W^2 |\vec{K}|^3} \int du u^2 |\vec{Q}| |\vec{q}^{(3)}| \left[ \frac{q_0^{(3)}}{|\vec{q}^{(3)}|} \log \frac{q_0^{(3)} + |\vec{q}^{(3)}|}{q_0^{(3)} - |\vec{q}^{(3)}|} - 2 \right] \sigma(u)$$

where, as before,  $k_\mu$ ,  $q_\mu^{(2)}$ , and  $q_\mu^{(3)}$  are the four-momenta of the photon, of  $\eta^0$ , and of  $\pi^+$ . In Eq. (8) they are taken in the reaction center of mass system (i. e. for which  $\vec{K} + \vec{p}_1 = 0$ ). The virtual pion momentum  $\vec{Q}$  is taken instead in the c. m. frame of the final  $\eta + n$ . As before we call  $W^2 = -(K+p_1)^2$  the squared energy in the reaction c. m. frame; and we have introduced  $u^2 = -(q_2 + p_2)^2$ , the squared  $\eta + n$  energy in the  $\eta + n$  c. m. frame. In Eq. (8) the cross-section is expressed as an integral of  $\sigma(u)$ , the cross-section for  $\pi + p \rightarrow \eta + n$ . We now assume that  $\sigma(u)$  is dominated by the  $N^{**}$  contribution and can be approximated by

$$(9) \quad \sigma(u) = \frac{2\pi}{|\vec{Q}|^2} \frac{\Gamma_\pi \Gamma_\eta}{(u - M^{**})^2 + (\Gamma^2/4)}$$

where  $M^{**}$  is the  $N^{**}$  mass,  $M^{**} \approx 1510$  MeV,  $\Gamma$  is its width that we take  $\sim 130$  MeV, and  $\Gamma_\pi$  and  $\Gamma_\eta$  are the partial widths for decay of  $N^{**}$  into  $p + \pi^-$  and into  $n + \eta^0$  respectively. Very little is known on these partial widths. Following Barkas and Rosenfeld<sup>(8)</sup>, we assume that the decay into  $p + \pi^-$  occurs in about 80% of the times. We write, for small values of  $|\vec{Q}|$ ,  $\Gamma_\pi = g_\pi^2 |\vec{Q}|^5$  and  $\Gamma_\eta = g_\eta^2 |\vec{Q}^{(2)}|^5$ , where  $\vec{Q}^{(2)}$  is  $\eta$  momentum in the  $\eta + n$  c. m. system. The "reduced width"  $g_\pi^2$ , with the above values of  $\Gamma$  and  $\Gamma_\pi/\Gamma$ ,

is  $\sim 2 \times 10^{-3} \text{ m}_\pi^{-4}$ . In the lack of better information, we assume that  $\sigma_\eta^2 = \sigma_\pi^2$ . We find  $\Gamma_\eta / \Gamma_\pi = 3 \times 10^{-3}$ . With these data we have computed the cross-section  $\sigma$  of Eq. (8). It is reported in fig. 2 (dotted line).

One sees from fig. 2 that the peripheral contribution of fig. 1 becomes larger, as the energy increases, than that from the other mechanism. Already at 2 BeV incident photon energy in the lab. system the peripheral mechanism through the  $\eta \rightarrow 2\pi + \gamma$  vertex comes out to be about ten times larger than the mechanism based on  $N^{**}$ . Unfortunately the figures we have used for  $N^{**}$  are rather tentative. The rather peculiar angular distribution of the peripheral mechanism of fig. 1, should possibly be used to identify the important production mechanism. In any case our calculations strongly support the possibility that a reliable measurement of the  $\eta^0$  lifetime may soon be possible from peripheral photoproduction of  $\eta + \pi^+$  on proton.

#### References

- (1) - P. L. Bastien et al. , Phys. Rev. Letters 8, 114 (1962).
- (2) - N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961).
- (3) - G. Puppi, Ann. Rev. Nuclear Sci. , 1963 (edited by E. Segré, Palo Alto, California), pag. 335.
- (4) - G. Bellettini, C. Bemporad, P. L. Braccini, L. Foà and M. Toller, Physics Letters 3, 170 (1963).
- (5) - A. Contogouris and A. Verganelakis, Physics Letters 6, 105 (1963).
- (6) - E. Celeghini and R. Gatto, Nuovo Cimento (to be published).
- (7) - Private communication by Professor R. Querzoli.
- (8) - A. Barkas and A. Rosenfeld, Lawrence Radiation Laboratory preprint.

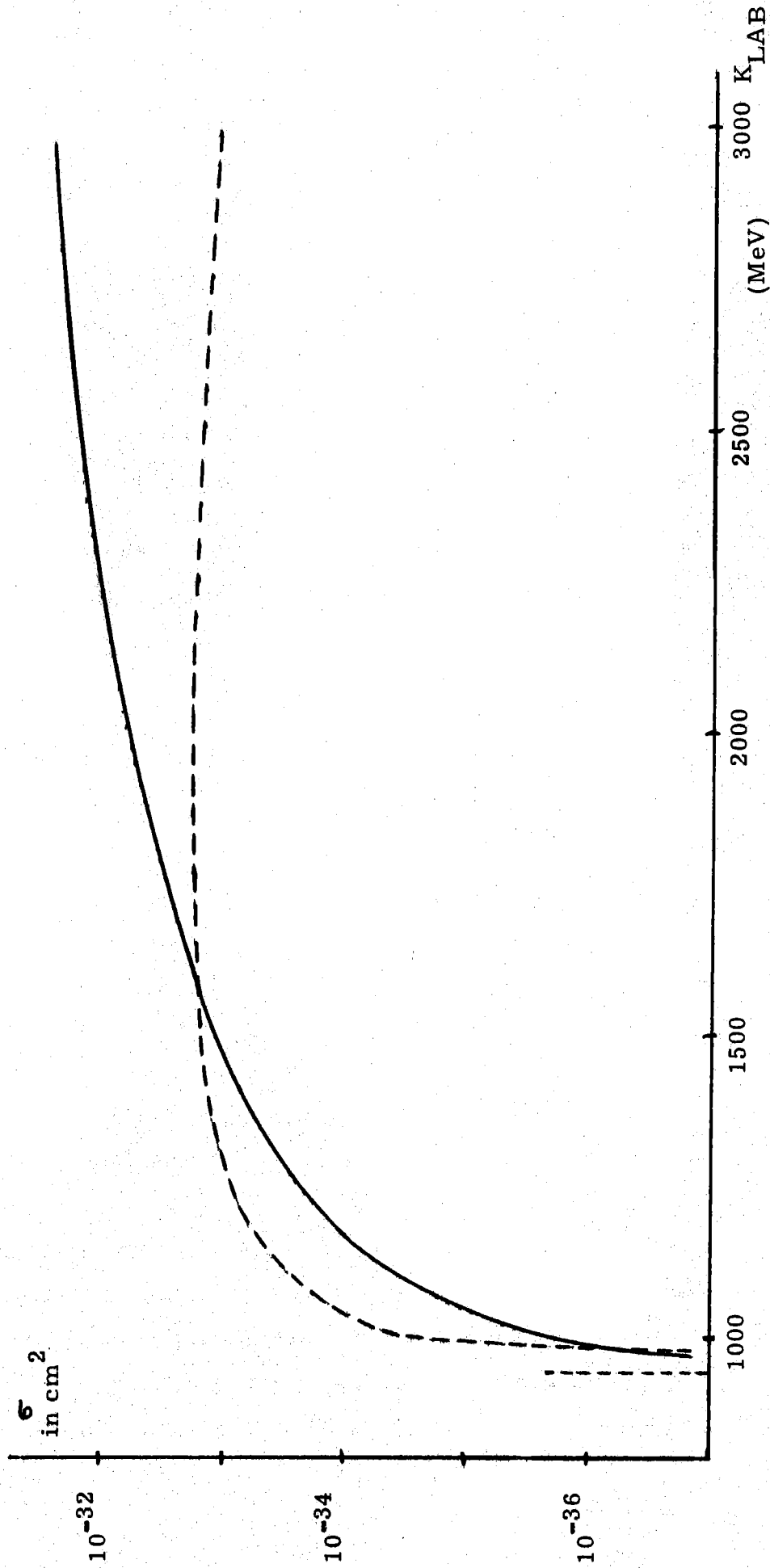


FIG. 2 - Full line: total cross-section for  $\gamma + p \rightarrow n + \gamma^0 + \pi^+$  calculated from the peripheral graph of fig. 1. The cross-section is calculated for a value of  $1.7 \times 10^{17} \text{ sec}^{-1}$  of  $w(\gamma^0 \rightarrow 2\theta)$  and for a value of 0.16 for  $R = (\gamma \rightarrow 2\pi\theta) / (\gamma \rightarrow 2\theta)$ . For larger  $w$  or  $R$  the cross-section must be proportionally increased. Dotted line: total cross-section for the same reaction from the peripheral graph of fig. 3. The elastic cross section is supposed to be dominated by the  $N_{\pi\pi}^+$  resonance.